A Vertex-Based Finite-Volume Time-Domain Method for Analyzing Waveguide Discontinuities

C. H. Chan and J. T. Elson

Abstract—**A vertex based finite-volume time-domain method is proposed for analyzing parallel-plate waveguide discontinuities. An irregular grid is chosen so that it can conform linearly to any discontinuity. In this scheme, the electric field vector is located at the vertices of the grid and the magnetic vector is located at the centroids of the elements formed by the grid. Reflection and transmission coefficients are calculated for the TE case. Excellent agreement is obtained for single- and double-step discontinuities when compared with the mode matching method.**

I. Introduction

The SIMPLICITY and usefulness of the finite-difference time-domain method [1] are widely accepted. However, a significant error due to the staircasing effect may occur when a curved boundary exists in the compuational domain. Various schemes have been developed to eradicate this difficulty [2]–[7]. All these methods of discretization can be considered to be edge-based in the sense that the variables are located at the middle of the edges of the grids. In this paper, a vertex based finite-volume time-domain (FVTD) method proposed recently [8], [9] is employed to analyze parallel-plate waveguide discontinuities. The computational domain is discretized with a triangular mesh so that any curved boundary can be approximated by piece-wise linear line segments. The electric field vector is sampled at the vertices of the triangles and the magnetic field vector is sampled at the centroids of the triangles. This method is not limited by the choice of triangulation and is more general than the triangulation presented in [10]. When using the Delaunay–Voronoi property for Delaunay triangulation, the location of the magnetic field in [10] will be equivalent to the present method. This new algorithm is stable even if the triangles are moderately distorted from the equilateral situation.

II. Vertex-Based Finite-Volume Time-Domain Method

To demonstrate how this vertex-based FVTD method works, let us consider the triangular grid depicted in Fig. 1. For the TE case, E_y is sampled at the vertices and both H_x and H_z are sampled at the centroids of the triangles. The electric and magnetic fields are assumed to be constant within the triangle of area A_e and the polygon of area A_h, respectively. The fields

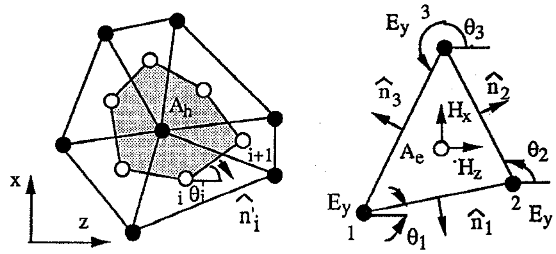

Fig. 1. A vertex-based finite-volume time-domain grid for a two-dimensional problem.

along an edge are assumed to be constant obtained from the averaged values of the fields at the two endpoints of the edge. With these assumptions, the FVTD equations given in (1) and (2) reduce to (3) and (4), respectively, for the two-dimensional problem:

$$\int \nabla \times \vec{E} dv = \int \left(\hat{n} \times \vec{E}\right) ds = -\mu \frac{\partial}{\partial t} \int \vec{H} dv, \quad (1)$$

$$\int \nabla \times \vec{H} dv = \int \left(\hat{n} \times \vec{H}\right) ds = \frac{\partial}{\partial t} \int \vec{E} dv, \quad (2)$$

$$\vec{H}^{n+\frac{1}{2}} = \vec{H}^{n-\frac{1}{2}} - \frac{\Delta t}{\mu A_e} \sum_{i=1}^{3} \hat{n}_i \times \left(\frac{\vec{E}_i^n + \vec{E}_{i+1}^n}{2}\right) L_i, \quad i+1 = 1 \text{ when } i = 3, \quad (3)$$

$$\vec{E}^{n+1} = \vec{E}^n + \frac{\Delta t}{\mu A_h} \sum_{i=1}^{N} \hat{n}'_i \times \left(\frac{\vec{H}_i^{n+\frac{1}{2}} + \vec{H}_{i+1}^{n+\frac{1}{2}}}{2}\right) L'_i, \quad i+1 = 1 \text{ when } i = N, \quad (4)$$

where $\Delta t < 2r/c$. Here, r is the radius of the smallest circle inscribed in the triangles or polygons, c is the velocity of light, and N is the number of sides of the polygon. The lengths of the ith edge of the triangle and polygon are L_i and L'_i, respectively. While a rigorous stability condition has not been derived, the time step chosen here is stable for all the examples shown in this paper.

For the TE case, E_y, H_x, and H_y are substituted into (3) and (4) from which three scalar equations can be obtained as follows:

$$H_x^{n+\frac{1}{2}} = H_x^{n-\frac{1}{2}} + \frac{\Delta t}{\mu A_e} \sum_{i=1}^{3} \frac{E_{yi}^n + E_{yi+1}^n}{2} L_i \sin\theta_i, \quad (5)$$

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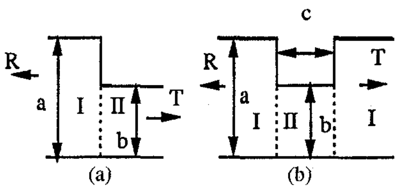

Fig. 2. Step discontinuity. (a) Single step; (b) Double step. $a = 0.8\lambda_o, b, = 0.6\lambda_o$ and $c = 0.2\lambda_o$.

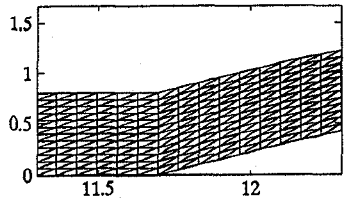

Fig. 3. Triangular mesh for the vertex based finite-volume time-domain grid.

$$H_z^{n+\frac{1}{2}} = H_z^{n-\frac{1}{2}} + \frac{\Delta t}{\mu A_e} \sum_{i=1}^{3} \frac{E_{yi}^n + E_{yi+1}^n}{2} L_i \cos\theta_i, \quad (6)$$

$$E_y^{n+1} = E_y^n + \frac{\Delta t}{\mu A_h} \sum_{i=1}^{N} \left(\frac{H_{xi}^{n+\frac{1}{2}} + H_{xi+1}^{n+\frac{1}{2}}}{2} \sin\theta_i' + \frac{H_{zi}^{n+\frac{1}{2}} + H_{zi+1}^{n+\frac{1}{2}}}{2} \cos\theta_i' \right) L_i'. \quad (7)$$

Time stepping these three equations yields the time-domain response of parallel-plate waveguides. The second-order Liao absorbing boundary condition [11] is enforced for the total and scattered E_y field at the terminal plane and the source plane, respectively. A detailed discussion on the application of the absorbing boundary condition on the scattered field in the total field formulation can be found in [12].

III. Numerical Results

To establish the accuracy of this new method, we calculate the reflection and transmission coefficients of single- and double-step parallel plate waveguide discontinuities (see Fig. 2) and compare the results with those obtained from the mode matching method. Excellent agreement is shown in Table I when compared to the mode matching method using 3 modes on either side of the discontinuity. A discretization of 15 points per free-space wavelength (λ_o) is chosen. In Table I, Z_a and Z_b are the characteristic impedances at Regions I and II, respectively. Next, we consider a discontinuity configuration that cannot be analyzed by the mode matching method. Fig. 3 shows a portion of the triangular discretization of the discontinuity which is described by a sine function. The scales shown in Fig. 3 are in terms of wavelengths. The contour plots of the electric field at various time steps within half a period are shown in Figures 4a to 4d. Here the scales are in terms of discretization $\Delta x = \Delta y = 0.067\lambda_o$. The magnitudes of the reflection and transmission coefficients are 0.102 and 0.993, respectively, giving a power check of 0.997. Although not implemented in this paper, this vertex based FVTD can be applied to the locally distorted grid in the vincinity of the discontinuity and blended smoothly into a rectangular grid for the conventional FDTD calculation in the region away from the arbitrary discontinuity.

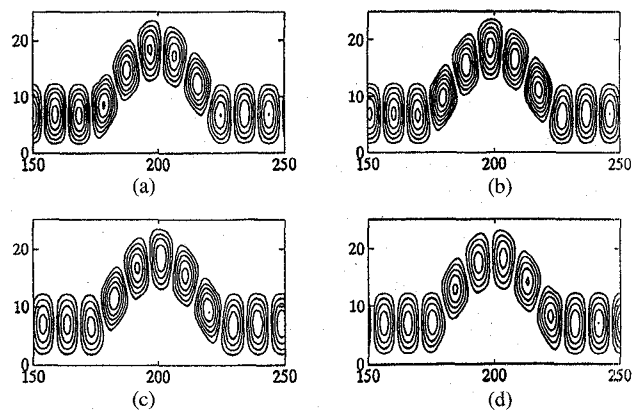

Fig. 4. Contour plots of fields propagating through a waveguide discontinuity described by a sine function. (a) At t; (b) At $t + T/8$; (c) At $t + T/4$; (d) At $t + 3T/8$.

TABLE I
Comparison of the Present Method and the Mode Matching Method
$Z_a = 482.95$ ohms and $Z_b = 682.02$ ohms

	$\|R\|$	$\|T\|$	Power Check
Fig. 2(a)			$\|R\|^2 + \|T\|^2(bZ_a)/(aZ_b)$
Mode Matching FVTD	0.156 0.157	1.355 1.343	1.000 0.983
Fig. 2(b)			$\|R\|^2 + \|T\|^2$
Mode Matching FVTD	0.298 0.308	0.955 0.948	1.000 0.997

IV. Conclusion

We have presented a vertex based FVTD method for analyzing arbitrary plate waveguide discontinuities. The primary grid is a triangular grid while the complementary grid consists of polygons formed by connecting the centroids of the triangles of the primary grid. Excellent agreement in the reflection and transmission coefficients is obtained when compared to the mode matching method for step discontinuities. The flexibility of the method in conforming to the shape of the waveguide discontinuity is demonstrated via the example of a sine function. A study on the extension of the method for treating three-dimensional problems is currently underway.

Acknowledgment

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